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# FINAL CA MAY '19 REVISION NOTES 

 Strategic Financial Management
## Derivatives :

Futures \& Options

## DERIVATIVES : FUTURES \& OPTIONS

## SECTION 1 : FUTURES

## Definition

A futures contract is a legal agreement, generally made on the trading floor of a futures exchange, to buy or sell a particular commodity or financial instrument at a predetermined price at a specified time in the future.

## Common types of futures:

Some of the most common types of futures traded across the globe are:
a. Stock Futures
b. Index Futures
c. Commodity Futures
d. Currency Futures.

## Advantages and Disadvantages of Futures

## Advantages

a. It allows hedgers to shift risks to speculators.
b. It gives traders an efficient idea of what the futures price of a stock or value of an index is likely to be.
c. Based on the current future price, it helps in determining the future demand and supply of the shares.
d. Since it is based on margin trading, it allows small speculators to participate and trade in the futures market by paying a small margin instead of the entire value of physical holdings.

## Disadvantages

However, one must be aware of the risks involved too.
a. The main risk stems from the temptation to speculate excessively due to a high leverage factor, which could amplify losses in the same way as it multiplies profits.
b. Further, as derivative products are slightly more complicated than stocks or tracking an index, lack of knowledge among market participants could lead to losses.
c. Since these are exchange traded products - they have standard lot sizes and expiry dates. This might pose a problem for matching the risk exposures in terms of quantity (or value) and time

## Pricing of Futures

The most commonly used model for pricing of futures is known as the "Cost of Carry model". According to cost of carry model the price of Futures $=$ Spot price + Net Cost of Carry. The formula can be modified slightly by expanding the term Net cost of carry as Cost of Carry - Carrying return.

In other words :
Future $=$ Spot $+($ Cost of carry - Carrying return)
Carry Cost refers to the cost of holding the asset till the futures contract matures. This could include storage cost, interest paid to acquire and hold the asset, financing costs etc.

Carry Return refers to any income derived from the asset while holding it like dividends, bonuses etc. While calculating the futures price of an index, the Carry Return refers to the average returns given by the index during the holding period in the cash market. A net of these two is called the net cost of carry.

Mathematically the basic pricing model can be expressed as follows :

## Futures $=\operatorname{Spot}(1+r)^{n}$

However, this basic model undergoes slight modification depending on the type of futures being calculate e.g stock futures, index futures or commodity futures. This modification is required to factor in storage costs (typically in case of commodities) and dividend returns (in case of stocks / index).

Hence we can state as under:

## Commodity futures:

Futures $=($ Spot + Present Value of Storage $)(1+\mathrm{r})^{\mathrm{n}}$

## Stock futures:

Futures $=($ Spot - Present Value of Dividends $)(1+r)^{\mathrm{n}}$

## Index futures

Futures $=\operatorname{Spot}[1+(r-d)]^{n}$

## What is continuous compounding

Continuous compounding is the mathematical limit that compound interest can reach. It is an extreme case of compounding since most interest is compounded on a monthly, quarterly or semiannual basis. Hypothetically, with continuous compounding, interest is calculated and added to the account's balance every infinitesimally small instant. While this is not possible in practice, the concept of continuously compounded interest is important in finance and is very frequently used in valuation of derivatives.
Continuous compounding can be expressed as:
Amount $=$ Principal x $\mathrm{e}^{\mathrm{rn}}$
Where $\mathrm{e}=2.7183$ (constant); $\mathrm{r}=$ continuously compounded rate of interest (CCRI) and $\mathrm{n}=$ time
If futures price is to be computed using a continuously compounded rate then the models can be expressed as under:

## Stock futures :

Futures $=($ Spot - PV of Dividend $) \times \mathrm{e}^{\mathrm{rn}}$

## Commodity futures:

Futures $=($ Spot +PV of Storage $) \mathrm{xe}^{\mathrm{rn}}$

## Index futures:

Futures $=\operatorname{Spot} X e^{(r-d) n}$

## Margins

Since futures are exchange traded products, they are subject to initial margins and mark to market margins (MTM margins).
Initial Margin : This is the margin required to be paid at the initiation of the trade. Normally it is at $10 \%$ of the trade value. Both the buyer as well as seller are required to pay the initial margins.

Mark to Market Margin (MTM Margin): Every day the trade in futures is marked to market and the resultant profit or loss is debited or credited to the clients account. This process is known as marking to market and the resultant debits or credits are called as Mark to Market margins.

Top up Margin : If the balance in the margin account goes below the maintenance levels (i.e the level which if breached would trigger the margin call) the client is required to bring in additional margins of such amount as is required to bring the balance in margin account back to the initial margin levels.

DISTINCTION BETWEEN FUTURES AND FORWARDS:

| Parameter | Forwards | Futures |
| :--- | :--- | :--- |
| Nature | A telephonic contract | An exchange traded product |
| Standardisation | Individually tailor made for every <br> customer and therefore has no <br> standard size | Since they are exchange traded - size <br> (lots) are standardised. |
| Settlement | Normally settled by actual <br> delivery | Cash settled - rarely settled by <br> delivery |
| Settlement | Settlement takes place between <br> two parties directly | Since they are exchange traded, <br> settlement takes place through <br> clearing houses |
| Transaction cost | Cost of forward contract is based <br> on bid- ask spread | Entails brokerage fees |
| Marking to market | No marking to market | Marked to market on a daily basis |
| Margins | No Margins required | Initial Margins to be put up at the time <br> of executing the trade |
| Credit risk | Subject to counter party risks | No counter party risks since <br> transaction happens through an <br> exchange and broker |

## SECTION 2: OPTIONS

## What are options?

In finance, an option is a contract which gives the buyer (the owner or holder of the option) the right, but not the obligation, to buy or sell an underlying asset or instrument at a specific strike price on a specified date, depending on the form of the option.

The seller has the corresponding obligation to fulfil the transaction-to sell or buy-if the buyer (owner) "exercises" the option.

An option that conveys to the owner the right to buy at a specific price is referred to as a call; an option that conveys the right of the owner to sell at a specific price is referred to as a put.

## DEFINITION OF VARIOUS TERMS USED IN OPTIONS

| Call option | Gives the buyer the right but not the obligation to buy the <br> underlying security at a specific price for a specified time. <br> The seller of the call option (writer) has the obligation to sell <br> the underlying asset if the buyer exercises his options |
| :--- | :--- |
| Put option | Gives the buyer the right but not the obligation to sell the <br> underlying security at a specific price for a specified time. <br> The seller of the call option (writer) has the obligation to buy <br> the underlying asset if the buyer exercises his options |
| Option <br> premium | Premium is the price at which the contract trades. <br> The premium is the price of the option and is paid by the <br> buyer of the option to writer or seller of the option. <br> The writer gains the premium irrespective of whether the <br> option is exercised or not. |
| Strike Price <br> /Exercise <br> price | It is the specified price at which the underlying asset is to be <br> bought or sold by the buyer if he exercises his option |
| Contract <br> Size | The number of shares of the underlying asset covered by the <br> options contract. |
| Open <br> Interest | Number of outstanding contract options in the exchange <br> market. <br> In the futures market it refers to the number of long or short <br> positions undertaken but not squared off. |
| American <br> option | An option which can be exercised at any time between the <br> date of purchase and the expiration date. |
| European <br> option | An option which can be exercised only on the expiration date. |
| Expiration <br> Date | The last day (in case of American option) or the only day in <br> case of European option on which the option can be exercised. <br> In India this date is the last Thursday (or previous business <br> day if Thursday is a holiday) of the expiration month |

At the money, In the money \& Out of the money
In the money
If by exercising the option the buyer makes a profit then the option is in the money

## At the money

If by exercising the option the buyer neither makes a profit nor a loss then the option is at the money

## Out of money

If by exercising the option the buyer makes a loss then the option is out of the money

## Option premium and its constituents

Option premium consists of two components: Intrinsic value + Time value

## Intrinsic value

- Intrinsic value is that part of option premium which represents the extent to which the option is in the money.
- The balance is the time value of money.
- An option which is out of money or at the money has zero intrinsic value.
- Intrinsic value can never be negative.


## Time Value

a. Also called as the extrinsic value of option
b. Represents the probability of the change in the underlying price that determines the value of the option during the remaining time till expiration.
c. This value depends on the time to expiration and the volatility of the underlying.
d. If an option is at the money or out of money the entire premium represents the time value.
e. Time value can also be never negative.
f. All options will have time value right upto the date they expire.
g. However as this time is constantly eroding, the time value of option declines over the balance period.
h. That is why options are referred to as wasting assets.
i. The decay begins slowly but starts accelerating towards the end causing the option to quickly lose value. This is because the market makers decide that it is unlikely that the underlying will gain value.

## Table showing Intrinsic Value

| If the option is | Does it have Intrinsic <br> Value | Does it have Time Value |
| :--- | :--- | :--- |
| IN THE MONEY | YES | YES |
| AT THE MONEY | NO | YES |
| OUT OF MONEY | NO | YES |

## Option spreads

Option spreads means taking position in two or more options of the same type (calls or puts) on the same underlying.

## Vertical spread [Price spread]

- Two legs have different strike price but the same expiration date.
- E.g. Buying a December Call with a strike price of Rs. 100 and sell a December Call on the same script with a strike price of Rs. 110 .


## Horizontal spread [Time spread]

- Two legs having the same strike price but different expiration date.
- E.g. Buying a December call option for Rs. 100 and selling a January Call option Rs. 100


## Diagonal spread [Price \& Time Spread]

- Two legs having different strike prices and different expiration dates.
- E.g. Buying a December call option for Rs. 100 and selling a December Call for Rs. 110


## Other option strategies

## Bull Put / Call Spread \& Bear Put / Call Spread

| Bull Call / Put spread | Bear Call put spread |
| :--- | :--- |
| Purchase and sale of put / call at <br> different strike price | Purchase and sale of put / call at different <br> strike price |
| Same Expiry date | Same Expiry date |
| Purchased put/ call to have a <br> lower strike price than the sold <br> put / call | Purchased put/ call to have a <br> higher strike price than the sold <br> put / call |
| Example <br> Purchase a Dec call at Rs.100 <br> and sell a Dec Call at Rs.110; <br> [Bull Call Spread] or | Example: <br> Purchase a Dec call at Rs.100 and sell a <br> Dec Call at Rs.90 [Bear Call Spread] |
| Purchase a Dec put at Rs.35 and <br> sell a Dec put at Rs.37 [Bull Put <br> Spread] | Purchase a Dec put at Rs.35 and sell a Dec <br> put at 33 [Bear Put Spread] |

## Straddle and Strangle

This strategy is adopted where the direction of the market is not known but the volatility is there in the underlying.
Long straddlle: purchasing a call and put option with the same exercise price.
Short straddlle: selling a call and put option with the same exercise price is a short straddle.
Long strangle: Purchasing a call and put option with different strike price
Short strangle: Selling a call and put option with different strike price

## Put Call Parity theorem

A portfolio comprising a call option and an amount of cash equal to present value of option's strike price has the same expiration value as a portfolio comprising a corresponding put option and the underlying.

If the expiration values of the two portfolios are the same then their present value must also be the same. This is called put call parity.

If two portfolios are going to have the same expiration value then they must have the same value today. If this is not the case then the investor can make an arbitrage profits.
Put call parity is not based on any option pricing model. It has been derived purely using arbitrage arguments

Let us make the following terminology for building a model:
$\mathrm{Sp}=$ Spot price today
P $=$ Price of Put
$\mathrm{C}=$ Price of call
X = Strike Price
St $=$ Market price on the strike day
Using the above Put Call parity theorem can be expressed as :
Put + Stock $=$ Call + Investment in Present Value of Strike Price ; i.e $\mathrm{P}+\mathrm{S}=\mathrm{C}+\mathrm{PV}(\mathrm{X})$

## Option Pricing Models

There are 3 main models which can be used for pricing options :

1) Binomial Riskless Model
2) Binomial Risk Neutral Model
3) Black Scholes Model

## Binomial Riskless Model

Binomial option pricing is a simple but powerful technique that can be used to solve many complex option-pricing problems. In contrast to the Black-Scholes and other complex option-pricing models that require solutions to stochastic differential equations, the binomial option-pricing model (two- state option-pricing model) is mathematically simple. It is based on the assumption of no arbitrage.
The assumption of no arbitrage implies that all risk-free investments earn the risk-free rate of return and no investment opportunities exist that require zero amount of investment but yield positive returns.

A model which ensures that the expiration values of any portfolio is the same irrespective of any price. Let us make the following assumptions:
i) CMP of stock = Rs. 100
ii) Possible moves $=$ Rs. 110 or Rs. 90
iii) Strike price of call option $=$ Rs. 100
iv) Price of Call = "C"
v) rate of interest $=6 \%$ CCRI
vi) Time to expiry $=1$ month

Our final objective is to find out the value of "C" i.e the option price

| Step I | Create a portfolio buying " $\mathrm{h} "$ <br> Rs. number of shares at the CMP of <br> Step 2 |
| :--- | :--- |
| Protect the above portfolio by selling a call at a price "C". Now the value <br> of the portfolio will be $\cdot .100 \mathrm{~h}-\mathrm{C}$ |  |
| THE IMPACT OF TWO POSSIBLE MOVES ARE ANALYSED BELOW |  |


| If stock price goes to ₹. 110 | If stock price goes to ₹ 90 |
| :---: | :---: |
| $\begin{gathered} \text { Portfolio Value }=\text { ₹ } 110 \mathrm{~h}-\mathrm{C} \text {; or } \\ \text { ₹ } 110 \mathrm{~h}-₹ 10 \\ \text { [Since Call would have a value of ₹ } 10 \text { ] } \end{gathered}$ | $\begin{aligned} \text { Portfolio Value } & =₹ 90 \mathrm{~h}-\mathrm{C} ; \text { or } \\ & ₹ 90 \mathrm{~h}-₹ 0 \end{aligned}$ <br> [Since Call would have a value of ₹ 0 ] |


| Step 3 | Since the values must be equal in a risk less model we can state that: <br> $₹ 110 \mathrm{~h}-₹ 10=₹ 90 \mathrm{~h}-₹ 0$ <br> Solving the above we get $\mathrm{h}=0.5$ |
| :--- | :--- |
| Step 4 | Substituting the value of h in $₹ 110 \mathrm{~h}-₹ 10$ or alternatively substituting the <br> value of h in $₹ 90 \mathrm{~h}-₹ 0$ we get the portfolio value on expiry as ₹ 45 |
| Step 5 | If the Value of portfolio on expiry is ₹ 45, then the value of portfolio today must <br> be equal to the Present Value of $₹ 45$ which is $₹ 45 \mathrm{x} \mathrm{e}^{-\mathrm{rt}}$ where $\mathrm{r}=6 \% \mathrm{CCRI}$ <br> and $\mathrm{t}=1 / 12$ |
| Step 6 | But we know that the value of portfolio today is ₹ $100 \mathrm{~h}-\mathrm{C}$ [refer step 2] |
| Step 7 | Hence we can say that $\cdot 100 \mathrm{~h}-\mathrm{C}=\cdot 45 \mathrm{x} \mathrm{e}^{-\mathrm{rt}}$ Solving the equation given that <br> $\mathrm{h}=0.5, \mathrm{e}=2.7183, \mathrm{r}=6 \% \mathrm{CCRI}$ and $\mathrm{t}=1 / 12$ we get the value of $\mathrm{C}=\mathrm{Rs} 5.22$ |

## Binomial Risk Neutral model

In the risk less hedge approach, the probability of the stock price increasing, Pu , or the probability of the stock price decreasing, $\mathrm{Pd}=1-\mathrm{Pu}$, did not enter into the analysis at all. In the risk neutral approach, given a stock price process (tree) we try to estimate these probabilities for a risk neutral individual and then use these risk neutral probabilities to price a call option.

In the above example for the Riskless model, the model ignored the probability of the prices hitting the level of Rs. 110 or Rs.90. The risk neutral valuation approach takes into consideration the probability of a stock price moving up or down and factors the same in calculating the price of an option.

## Example explaining the concept

Let $P_{u}$ be the probability of price going up; then the probability of price going down i.e $P_{d}=1-P_{u}$ Let us make the following assumption:

| i. CMP | $=$ | Rs. 75 |
| :--- | :--- | :--- | :--- |
| ii. Upward possible price | $=$ | Rs. 95 |
| iii. Downward possible price | $=$ | Rs. 63 |
| iv. Call option strike price | $=$ | Rs. 65 (one month) |
| v. Risk free rate | $=$ | $6 \%$ |

We know that : CMP of Stock $=$ Present Value of Future expected values
i.e CMP $=P V$ of [(upward price $\left.\times p_{u}\right)+\left(\right.$ downward price $\left.\left.\times p_{d}\right)\right]$
i.e CMP $=P V$ of [(upward price $\left.\times p_{u}\right)+\left(\right.$ downward price $\left.x\left(1-p_{u}\right)\right]$

Hence Rs. 75 today $=$ PV of expected values
Now,
Expected values $=P_{u}(95)+P_{d}(63)$; or
$\mathrm{P}_{(\mathrm{u})}(95)+\left(1-\mathrm{P}_{\mathrm{u}}\right)(63)$
PV of expected values $=\left[\mathrm{P}_{(\mathrm{u})}(95)+\left(1-\mathrm{P}_{\mathrm{u}}\right)(63)\right]^{*} \mathrm{e}^{-\mathrm{rt}}$
i.e Rs. $75=\left[\mathrm{P}_{(\mathrm{u})}(95)+\left(1-\mathrm{P}_{\mathrm{u}}\right)(63)\right]^{*} \mathrm{e}^{-\mathrm{rt}}$

Solving the above equation for $\mathrm{r}=6 \%$ and $\mathrm{T}=1$ month we get the values of $\mathrm{P}_{\mathrm{u}}$ and $\mathrm{P}_{\mathrm{d}}$
$\mathrm{P}_{\mathrm{u}}=0.38675$ and $\mathrm{P}_{\mathrm{d}}=(1-0.38675)$ i.e. 0.61325
Now we calculate the expected value of Call

| If price of stock on <br> expiry | Value of Call with <br> strike price $=\cdot 65$ | Probability | Expected value |
| :--- | :--- | :--- | :--- |
| 95 | 30 | 0.38675 | 11.6025 |
| 63 | 0 | 0.61325 | 0 |
| Expected value of call on expiry | 11.6025 |  |  |
| Hence value of call today = Present value of $11.6025 @ 6 \%$ per annum <br> for 1 month $: ₹ 11.6025 \mathrm{xe}^{-\mathrm{rt}}$ | $₹ 11.54$ |  |  |

## Black Scholes Model

a) First propogated in 1973 by Fischer Black and Myron Scholes.
b) It has become the standard for valuing options
c) Predominantly used for calculating European Options
d) It utilizes the stock price, strike price, expiration date, risk free return, and the standard deviation (volatility) of the stock's return.
The formula for Black Scholes model is:
$\mathrm{C}=\mathrm{S} . \mathrm{N} .\left(\mathrm{d}_{1}\right)-\mathrm{Xe}^{-\mathrm{rt}} . \mathrm{N}\left(\mathrm{d}_{2}\right)$
Where
C = Value of Option
S = Spot price
$\mathrm{X}=$ Strike price
$\mathrm{R}=$ risk free interest rate
$\mathrm{T}=$ time till expiration
$\mathrm{N}=$ area under normal curve
$\mathrm{D}_{1}=\left[(\log \mathrm{s} / \mathrm{x})+\left(\mathrm{r}+{ }^{2} / 2\right) \mathrm{T}\right] \mathrm{T}^{1 / 2}$
$\mathrm{D}_{2}=\mathrm{D} 1-\mathrm{T}^{1 / 2}$

## Valuing Real Options

Real Options Valuation, also often termed real options analysis,(ROV or ROA) applies option valuation techniques to capital budgeting decisions.A real option itself, is the right - but not the obligation - to undertake certain business initiatives, such as deferring, abandoning, expanding, staging, or contracting a capital investment project. For example, the opportunity to invest in the expansion of a firm's factory, or alternatively to sell the factory, is a real call or put option, respectively.
Real options are generally distinguished from conventional financial options in that they are not typically traded as securities, and do not usually involve decisions on an underlying asset that is traded as a financial security. A further distinction is that option holders here, i.e. management, can directly influence
the value of the option's underlying project; whereas this is not a consideration as regards the underlying security of a financial option. Moreover, management cannot lookup for a volatility as uncertainty, instead their perceived uncertainty matters in real options reasoning's. Unlike financial options, management also have to create or discover real options, and such creation and discovery process comprises an entrepreneurial or business task. Real options are most valuable when uncertainty is high; management has significant flexibility to change the course of the project in a favorable direction and is willing to exercise the options.

## Option Greeks

Gamma
Gamma is a measure of rate of change of delta for small changes in underlying stock price - in other words it is delta of the delta. While using delta to hedge the portfolio we need to keep in mind that we should keep the gamma very low. If gamma is very high then a small change in the underlying would result in delta going haywire and therefore your hedge might collapse.

## Theta

Theta measures the change in option price if the period to maturity reduces by one day. In simple words it is a measure of time decay. Now we know that time decay of an option is inevitable and bound to take place. So there is no point in hedging for the same.

All options - both Calls and Puts lose value as the expiration approaches. The Theta or time decay factor is the rate at which an option loses value as time passes. Theta is expressed in points lost per day when all other conditions remain the same. Time runs in one direction, hence theta is always a positive number, however to remind traders it's a loss in options value it is sometimes written as a negative number. A Theta of -0.5 indicates that the option premium will lose -0.5 points for every day that passes by. For example, if an option is trading at Rs.2.75/- with theta of -0.05 then it will trade at Rs.2.70/- the following day (provided other things are kept constant). A long option (option buyer) will always have a negative theta meaning all else equal, the option buyer will lose money on a day by day basis. A short option (option seller) will have a positive theta. Theta is a friendly Greek to the option seller. Remember the objective of the option seller is to retain the premium. Given that options loses value on a daily basis, the option seller can benefit by retaining the premium to the extent it loses value owing to time. For example if an option writer has sold options at Rs.54, with theta of 0.75 , all else equal, the same option is likely to trade at $-=0.75 * 3=2.25=54-2.25=51.75$ Hence the seller can choose to close the option position on T+ 3 day by buying it back at Rs.51.75/- and profiting Rs.2.25 ...

## Rho

Rho measures the change in option price given a one percentage change in risk free interest rate. In other words it measures how sensitive the option value is to change in interest rates. For example a Rho of 0.05 indicates that the options theoretical value will increase by 0.05 if interest rate is decreased by 1

Vega
Vega indicates the change in value of option for one percentage change in volatility. The option's vega is a measure of the impact of changes in the underlying volatility on the option price. Specifically, the vega of an option expresses the change in the price of the option for every $1 \%$ change in underlying volatility.

## Example of Vega

A stock XYZ is trading at • 46 in May and a JUN 50 call is selling for • 2. Let's assume that the vega of the option is 0.15 and that the underlying volatility is $25 \%$.
If the underlying volatility increased by $1 \%$ to $26 \%$, then the price of the option should rise to $\cdot 2+0.15$ $=$ - 2.15 .
However, if the volatility had gone down by $2 \%$ to $23 \%$ instead, then the option price should drop to ${ }^{-} 2$ $-(2 \times 0.15)=\$ 1.70$
Q. 1 Spot price of Infosys is ₹ 2000 . Interest rate prevailing is $14 \%$ per annum. Expected dividend after 2 months is ₹ 10 per share. Calculate what should be the expected price of Infosys today in the 3 months futures markets.
Q. 2 For X Ltd, spot rate $=₹ 70$, continuous compounded rate of interest is $8 \%$. Calculate price of future with 3 months expiry if the stock pays a dividend of $₹ 1.5$ on expiry [ $\mathrm{e} 0.02=1.02020$ ].
Q. 3 Stock index currently stands at ₹ 3500 . The risk free interest rate is $8 \%$ per annum \& the dividend yield on the index is $4 \%$ per annum. Calculate the 4 month index future if the of $8 \%$ is CCRI [ $\mathrm{e} 0.0133=1.014]$.
Q. 4 The current price of cotton is ₹ 400 per bale. The storage cost is ₹ 100 per bale per year payable in arrears. Assuming that interest rates are $10 \%$ per annum [CCRI], calculate the one year future price per bale of cotton [ $\mathrm{e} 0.10=1.1051]$.
Q. 5 On 31-7-2017 the value of stock index is 2600 . The risk free return is $9 \%$ per annum. The dividend yield on this stock index is as follows :

| Month | Dividend yield |
| :--- | :---: |
| January | $2 \%$ |
| February | $5 \%$ |
| March | $2 \%$ |
| April | $2 \%$ |
| May | $5 \%$ |
| June | $2 \%$ |
| July | $5 \%$ |
| August | $2 \%$ |
| September | $2 \%$ |
| October | $5 \%$ |
| November | $2 \%$ |
| December |  |

Assuming that interest is continuously compounded, what will be the future price of contract deliverable on 31-12-2011. Given e $0.02417=1.02446$ or say 1.0245
Q. 6 The following data relates to ABC Ltd's share prices
$\begin{aligned} \text { Current price per share } & =₹ 180 \\ \text { Price per share in the futures markets }-6 \text { months } & =₹ 195\end{aligned}$
Price per share in the futures markets - 6 months $=₹ 195$
It is possible to borrow money in the market for securities transaction at the rate of $12 \%$ per annum.

## Required:

a. Calculate the theoretical minimum price of 6 months futures.
b. Explain if any arbitrage opportunities exist.

## Margins on Futures

Q. 7 On November 15, when the spot price for TELCO is • 473 per share, Mr X buys 15 contracts of July TELCO futures at - 491. Assume that the initial margin for TELCO futures is - 800 per contract, and the maintenance margin is - 600 per contract. Given that each contract is 50 shares. Daily settlement prices for the next few days are as follows :
Nov $15^{\text {th }}$

- 496

Nov $16{ }^{\text {th }}$

- 503

Nov 17 ${ }^{\text {th }}$

- 488

Nov $18^{\text {th }}$

- 485

Nov $19^{\text {th }}$

- 491

Assume that Mr X withdraws profits from his margin account only once on Nov $16^{\text {th }}$ when he withdraws half the maximum amount allowed. Compute the balance in the account at the end of each of these days. Find his profit or loss at the end of Nov $19^{\text {th. }}$

## Futures \& Hedging

Q. 8 Ram buys 10000 shares of X Ltd at • 22 and obtains a complete hedge of shorting 400 Nifties at ₹ 1100 each. He closes out his position at the closing price of the next day at which point the share price of X Ltd has dropped $2 \%$ and the Nifty futures has dropped $1.5 \%$. What is the overall profit / loss on this set of transaction.
Q. 9 BSE Index

Value of Portfolio
Risk free interest rate
Dividend yield on Index
Beta of portfolio

## 5000

₹. 10,10,000
$9 \%$ per annum
$6 \%$ per annum
1.5

We assume that a futures contract on the BSE index with 4 months maturity is used to hedge the value of portfolio over next 3 months. One future contract is for delivery of 50 times the index. Based on the above information, calculate:
a. Price of future contract.
b. The gain on short position of futures if index turns out to be 4500 in 3 months
Q. 10 A company is long on 10 MT of copper at $₹ 474$ per kg (spot) and intends to remain so for the ensuing quarter. The standard deviation of change of its spot and future prices are $4 \%$ and $6 \%$ respectively, having a co-relation co-efficent of 0.75 . What is the hedge ratio? What is the amount of the copper futures it should short to achieve a perfect hedge.
Q. 11 A High Networth Individual (HNI) is holding the following portfolio is Rupees crores:

Investment in diversified equity shares
Cash and Bank Balance
Total
80.00
20.00

The Beta of the portfolio is 0.8 . The index futures is selling at 5500 levels. The HNI wants to increase the beta of the portfolio for he believes tha the market would up from the current level. How many index futures he should buy / sell so that the beta is increased to 1.20 . One index futures consists of 100 units.
Q. 12 On April 1, 2015 an investor has a portfolio consisting of eight securities as shown below:

The cost of capital for the investor is $20 \%$ per annum continuously compounded. The investor fears a fall in the prices of the shares in the near future. Accordingly he approaches you for the advice to protect the interest of his portfolio. You can make use of the following information:
a. The current NIFTY value is 8500
b. NIFTY futures can be traded in units of 25 only.
c. Futures for May are currently quoted at 8,700 and futures for June are being quoted at 8850 .

You are required to calculate :
i. The beta of his portfolio.
ii. The theoretical value of the futures contracts expiring in May and June
iii. Given that $\mathrm{e} 0.03=1.03045, \mathrm{e} 0.04=1.04081, \mathrm{e} 0.05=1.05127$
iv. The number of NIFTY contracts that he would have to sell if desires to hedge until June in each of the following cases:
a. His total portfolio
b. $50 \%$ of his portfolio
c. $120 \%$ of his portfolio

| Security | Market <br> Price | No of shares | Beta value |
| :--- | ---: | ---: | ---: |
| A | 29.40 | 400 | 0.59 |
| B | 318.70 | 800 | 1.32 |
| C | 660.20 | 150 | 0.87 |
| D | 5.20 | 300 | 0.35 |
| E | 281.90 | 400 | 1.16 |
| F | 275.40 | 750 | 1.24 |
| G | 514.60 | 300 | 1.05 |
| H | 170.50 | 900 | 0.76 |

ITM / ATM / OTM / Time value \& Intrinsic Value
Q. 13 Given the following data determine the value of the call option at their expiration dates:

| Option | Market price per <br> share on Expiry | Exercise price of <br> the option |
| :---: | :---: | :---: |
| A | 10 | 12 |
| B | 25 | 21 |
| C | 48 | 52 |
| D | 7 | 5 |

Q. 14 State whether each one of the following is In the Money (ITM), At the Money (ATM) or Out of Money (OTM)

| Option | Exercise price | Spot |
| :---: | :---: | :---: |
| Call | 60 | 55 |
| Call | 50 | 50 |
| Call | 110 | 105 |
| Call | 40 | 35 |
| Put | 110 | 100 |
| Put | 105 | 115 |
| Put | 12 | 15 |
| Put | 25 | 20 |

Q. 15 A stock with a current market price of ₹ 50 has the following exercise price and call option premium. Compute the intrinsic value and time value:

| Exercise price | 45 | 48 | 50 | 52 | 55 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Premium | 9 | 6 | 4 | 3 | 2 |

Q. 16 A stock with a current market price of ₹ 50 has the following exercise price and put option premium. Compute the intrinsic value and time value:

| Exercise price | 45 | 48 | 50 | 52 | 55 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Premium | 1 | 2 | 3 | 5 | 7 |

## Pay off tables / Spreads / Strategies

Q. 17 The equity shares of Ramacast Ltd are being sold at ₹ 210 . A 3 month call option is available for a premium of ₹ 6 per share and a 3 month Put is available for a premium of ₹ 5 per share. Find out the net pay off of the option holder of the call option and put option given that (i) the strike price in both the case is ₹ 220 , and (ii) the share price on expiry is ₹ 200 or ₹ 210 or₹ 220 or ₹ 230 or ₹ 240
Q. 18 Equity shares of Casio Ltd are being currently sold for ₹ 90 per share. Both the call option and the put option for 3 month period are available for a strike price of ₹ 97 at a premium of ₹ 3 and ₹ 2 respectively. Prepare the payoff table if price of share on expiry ranges between 80 to ₹ 120 in ticks of ₹ 10 if an creates a (i) Strip or (ii) Strap
Q. 19 A trader buys for ₹ 3 a call with a strike price of $₹ 30$ and sells for $₹ 1$ a call with a strike price of $₹$ 35. Calculate his net pay off if the stock price at the end of expiration period is:
a. Less than or equal to ₹ 30
b. More than or equal to ₹ 35
c. About ₹ 30 but below ₹ 35
Q. 20 A trader buys for ₹ 1 a call with a strike price of $\cdot 35$ and sells for • 3 a call with a strike price of ₹ 30. Calculate his net pay off if the stock price at the end of expiration period is:
a. Less than or equal to ₹ 30
b. More than or equal to ₹ 35
c. About ₹ 30 but below ₹ 35
Q. 21 Suppose that a certain stock is currently worth ₹ 61 . A trader feels that a significant price move in the next 6 months is unlikely. The market price of 6 month calls are as follows:

| Strike Price (₹) | Call Price (₹) |
| :--- | :--- |
| 55 | 10 |
| 60 | 7 |
| 65 | 5 |

Create a Butterfly spread for the investor and show the payoff of the overall strategy presuming that price of share on expiry could range between ₹ 30 to ₹ 80 in ticks of ₹ .5

## Put Call Parity / Arbitrage

Q. 22 You are given the following information about ABC Ltd's share and call option:

| Current share price | $₹ 90$ |
| :--- | :--- |
| Option Strike Price | $₹ 110$ |
| Risk free interest rate | $10 \%$ |
| Time to option expiry | 1 year |
| Call Premium | 12 |
| Put Premium | 21 |

You are required to analyses if there is any arbitrage opportunity
Q. 23 You are given the following information about ABC Ltd's share and call option:

| Current share price | $₹ 90$ |
| :--- | :--- |
| Option Strike Price | $₹ 110$ |
| Risk free interest rate | $10 \%$ |
| Time to option expiry | 1 year |
| Call Premium | 12 |
| Put Premium | 24 |

You are required to analyses if there is any arbitrage opportunity

## Option valuation: Binomial / Black Scholes

Q. 24 Consider a two year American call option with a strike price of ₹ 50 on a stock the current price of which is also - 50 . Assume that there are two time periods of one year and in each year the stock price can move up or down by equal percentage of $20 \%$. The risk free interest rate is $6 \%$. Using Binomial risk neutral model calculate the value of call and probability of price moving up and down. Also draw a two-step binomial tree showing prices and payoffs at each.
Q. 25 An equity share is currently selling for Rs. 80. In a year's time. It can rise by 30 percent or fall by 15 percent. The exercise price of a call option on this share is Rs. 90 .
What is the value of the call option if the risk-free rate is 8 percent? Use the Binomial Method risk less model
Q. 26 X Ltd’s share is currently trading at ₹ 220. It is expected that in 6 months' time it could double or halve. One year call option on X Ltd' share has an exercise price of ₹ 165 . Assuming the risk free rate of interest to be $20 \%$ calculate:
(a) value of call option of X Ltd's share.
(b) Option delta for the second 6 month in case the stock price rises to ₹ 440 or falls to ₹ 110
(c) Now suppose in 6 months the share price is ₹ 110 . How at this point can we replicate portfolio of call options and risk free lending.
Q. 27 (i) The shares of TIC Ltd are currently priced at - 415 and call option exercisable in three months' time has an exercise rate of ₹ 400 . Risk free interest rate is $5 \%$ p.a (CCRI) and standard deviation of share price is $22 \%$. Based on the assumption that TIC is not going to declare any dividend over the next 3 months is the option worth buying at ₹ 25 .
(ii) Calculate the aforesaid call option based on Black Scholes Model if the current market price of share is ₹ 380
(iii) What would be the worth of Put option if the current price is considered ₹ 380 .
(iv) If TIC share price at present is taken at - 408 and a dividend of $₹ 10$ is expected to be paid in 2 months' time then calculate the value of call option.
Q. 28 From the following data for a certain stock, find the value of call option:

Price of stock now
$=$ ₹ 80
Exercise price $=$ ₹ 75
Standard Deviation
Maturity period
Annual Interest rate
Given :

| No of S.D from mean (z) | Area of the left or right (one tail) |
| :---: | :---: |
| 0.25 | 0.4013 |
| 0.30 | 0.3821 |
| 0.55 | 0.2912 |
| 0.60 | 0.2578 |

$e^{0.06}=1.060$ and $\operatorname{Ln} 1.0667=0.0645$
Q. 29 You are trying to value a long term call option on the Standard and Poor 500, expiring in 2 months with a strike price of $\$ 900$. The index is currently at $\$ 930$ and the annualized standard deviation in stock prices is $20 \%$ per annum. The average dividend yield on the index is $0.3 \%$ per month, and expected to remain unchanged over the next month. The treasury bond rate is $8 \%$.
a. Estimate the value of the long term call option.
b. Estimate the value of a put option with the same parameters.
Q. 30 IPL is already in Production of Fertiliser and is considering a proposal of building a new plant to produce pesticides. Suppose the Present value of proposal is - 100 crores without the abandonment option. However, if market conditions for pesticide turns out to be favourable the PV of the proposal shall increase by $30 \%$. On the other hand if the market conditions remain sluggish the PV of the proposal shall be reduced by $40 \%$. In case company is not interested in continuation of the project it can be disposed off for ₹ 80 crores. If the risk free rate of interest is $8 \%$ then what will be the value of abandonment option.
Q. 31 ABC Ltd is a pharmaceutical company possessing a patent of a drug called "Aidrex" a medicine for aids patients. Being an approach drug ABC Ltd holds the right of production of drugs and its marketing. The period of the patent is 15 year after which any other pharmaceutical company can produce the drug with the same formula. It is estimated that company shall required to incur \$ 12.5 million for development and marketing of the drug. As per survey conducted the present value of expected cash flow from the sale of drug during 15 years shall be $\$ 16.70$ million. Cash flow from the previous similar type of drug have exhibited a variance of $26.8 \%$ of the present value of cash flows. The current yield on Treasury Bonds of similar duration (15 years) is $7.8 \%$. Determine the value of the patent.

## Arbitrage between Futures and Options

Q. 41 You are given the following information about ABC's futures and options:

March futures price
Details of March options are as under:
Strike Price
$=₹ 150$
Call Premium
=₹ 20
Put Premium
$=₹ 50$
Explain if any arbitrage opportunity exists
Q. 42 You are given the following information about ABC's futures and options:

March futures price
Details of March options are as under:
Strike Price
Call Premium
Put Premium
Explain if any arbitrage opportunity exists

TABLE : AREAS UNDER THE STANDARD NORMAL CURVE FROM 0 TO Z.

| Z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| .0 | .0000 | .0040 | .0080 | .0120 | .0160 | .0199 | .0239 | .0279 | .0319 | .0359 |
| .1 | .0398 | .0438 | .0478 | .0517 | .0557 | .0596 | .0636 | .0675 | .0714 | .0753 |
| .2 | .0793 | .0832 | .0871 | .0910 | .0948 | .0987 | .1026 | .1064 | .1103 | .1141 |
| .3 | .1179 | .1217 | .1255 | .1293 | .1331 | .1368 | .1406 | .1443 | .1480 | .1517 |
| .4 | .1554 | .1591 | .1628 | 1664 | .1700 | .1736 | .1772 | .1808 | .1844 | .1879 |
| .5 | .1915 | .1950 | .1985 | .2019 | .2054 | .2088 | .2123 | .2157 | .2190 | .2224 |
| .6 | .2257 | .2291 | .2324 | .2357 | .2389 | .2422 | .2454 | .2486 | .2518 | .2549 |
| .7 | .2580 | .2612 | .2642 | .2673 | .2704 | .2734 | .2764 | .2794 | .2823 | .2852 |
| .8 | .2881 | .2910 | .2939 | .2967 | .2995 | .3023 | .3051 | .3078 | .3106 | .3133 |
| .9 | .3159 | .3186 | .3212 | .3238 | .3264 | .3289 | .3315 | .3340 | .3365 | .3389 |
| 1.0 | .3413 | .3438 | .3461 | .3485 | .3508 | .3531 | .3554 | .3577 | .3599 | .3621 |
| 1.1 | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| 1.2 | .3849 | .3869 | .3888 | .3907 | .3925 | .3944 | .3962 | .3980 | .3997 | .4015 |
| 1.3 | .4032 | .4049 | .4066 | .4082 | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |
| 1.4 | .4192 | .4207 | .4222 | .4236 | .4251 | .4265 | .4279 | .4252 | .4306 | .4319 |
| 1.5 | .4332 | .4345 | .4357 | .4370 | .4382 | .4394 | .4406 | .4418 | .4429 | .4441 |
| 1.6 | .4452 | .4463 | .4474 | .4484 | .4495 | .4505 | .4515 | .4525 | .4535 | .4545 |
| 1.7 | .4554 | .4564 | .4573 | .4582 | .4591 | .4599 | .4608 | .4616 | .4625 | .4633 |
| 1.8 | .4641 | .4649 | 4656 | .4664 | .4671 | .4678 | .4686 | .4693 | .4699 | .4706 |
| 1.9 | 4713 | .4719 | .4726 | .4732 | .4738 | .4744 | .4750 | .4756 | .4761 | .4767 |
| 2.0 | 4772 | .4778 | .4783 | .4788 | .4793 | .4798 | .4803 | 4808 | .4812 | .4817 |
| 2.1 | .4821 | 4826 | .4830 | .4834 | .4838 | .4842 | .4846 | .4850 | .4854 | .4857 |
| 2.2 | 4861 | 4864 | .4868 | .4871 | .4875 | .4878 | .4881 | .4884 | .4887 | 4890 |
| 2.3 | .4893 | .4896 | .4898 | .4901 | .4904 | .4906 | .4909 | .4911 | .4913 | .4916 |
| 2.4 | .4918 | .4920 | .4922 | .4925 | .4927 | .4931 | .4931 | .4932 | .4934 | .4936 |
| 2.5 | .4938 | .4940 | .4941 | .4943 | .4945 | .4946 | .4948 | .4949 | .4951 | .4952 |
| 2.6 | .4953 | .4955 | .4956 | .4957 | .4959 | .4960 | .4961 | .4962 | .4963 | .4964 |
| 2.7 | 4965 | .4966 | .4967 | .4968 | .4969 | .4970 | .4971 | .4972 | .4973 | .4974 |
| 2.8 | .4974 | .4975 | .4976 | .4977 | .4977 | .4978 | .4979 | .4979 | .4980 | .4981 |
| 2.9 | .4981 | .4982 | .4982 | .4983 | .4984 | .4984 | .4985 | .4985 | .4986 | .4986 |
| 3.0 | .49865 | .4987 | .4987 | .4988 | .4988 | .4989 | .4989 | .4989 | .4990 | .4990 |
| 4.0 .4999683 |  |  |  |  |  |  |  |  |  |  |

Illustration: For $Z=1.72$, shaded area is .4573 out of total area of 1 .


| $\mathbf{x}$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | . 5000 | . 5040 | 5080 | . 5120 | . 5160 | . 5199 | . 5239 | . 5279 | . 5319 | . 5359 |
| 0.1 | . 5398 | . 5438 | . 5478 | . 5517 | . 5557 | . 5596 | . 5636 | . 5675 | . 5714 | . 5754 |
| 0.2 | . 5793 | . 5832 | . 5871 | . 5910 | . 5948 | . 5987 | . 6026 | . 6064 | . 6103 | . 6141 |
| 0.3 | . 6179 | . 6217 | . 6255 | . 6293 | . 6331 | . 6368 | . 6406 | . 6443 | . 6480 | . 6517 |
| 0.4 | . 6554 | . 6591 | . 6628 | . 6664 | . 6700 | . 6736 | . 6772 | . 6808 | . 6844 | . 6879 |
| 0.5 | . 6915 | . 6950 | . 6985 | . 7019 | . 7054 | . 7088 | . 7123 | . 7157 | . 7190 | . 7224 |
| 0.6 | . 7258 | . 7291 | . 7324 | . 7357 | . 7389 | . 7422 | . 7454 | . 7486 | . 7518 | . 7549 |
| 0.7 | . 7580 | . 7910 | . 7939 | . 7967 | . 7996 | . 8023 | . 8051 | . 8078 | . 8106 | . 8133 |
| 0.9 | . 8159 | . 8186 | . 8212 | . 8238 | . 8264 | . 8289 | . 8315 | . 8340 | . 8365 | . 8389 |
| 1.0 | . 8413 | . 8438 | . 8461 | . 8485 | . 8508 | . 8531 | . 8554 | . 8577 | . 8599 | . 8621 |
| 1.1 | . 8643 | . 8665 | . 8686 | . 8708 | . 8729 | . 8749 | . 8770 | . 8790 | . 8810 | . 8830 |
| 1.2 | . 8849 | . 8869 | . 8888 | . 8907 | . 8925 | . 8944 | . 8962 | . 8980 | . 8997 | . 9015 |
| 1.3 | . 9032 | . 9049 | . 9066 | . 9082 | . 9099 | . 9115 | . 9131 | . 9147 | . 9162 | . 9177 |
| 1.4 | . 9192 | . 9207 | . 9222 | . 9236 | . 9251 | . 9265 | . 9279 | . 9292 | . 9306 | . 9319 |
| 1.5 | . 9332 | . 9345 | . 9357 | . 9370 | . 9382 | . 9394 | . 9406 | . 9418 | . 9429 | . 9441 |
| 1.6 | . 9452 | . 9463 | . 9474 | . 9484 | . 9495 | . 9505 | . 9515 | . 9525 | . 9535 | . 9545 |
| 1.7 | . 9554 | . 9564 | . 9573 | . 9582 | . 9591 | . 9599 | . 9608 | 9616 | . 9625 | . 9633 |
| 1.8 | . 9641 | . 9649 | . 9656 | . 9664 | . 9671 | . 9678 | . 9686 | . 9693 | . 9699 | . 9706 |
| 1.9 | . 9713 | . 9719 | . 9726 | . 9732 | . 9738 | . 9744 | . 9750 | . 9756 | . 9761 | . 9767 |
| 2.0 | . 9772 | . 9778 | . 9783 | . 9788 | . 9793 | . 9798 | . 9803 | . 9808 | . 9812 | . 9817 |
| 2.1 | . 9821 | . 9826 | . 9830 | . 9834 | . 9838 | . 9842 | . 9846 | . 9850 | . 9854 | . 9857 |
| 2.2 | . 9861 | . 9864 | . 9868 | . 9871 | . 9875 | . 9878 | . 9881 | . 9884 | . 9887 | . 9890 |
| 2.3 | . 9893 | . 9896 | . 9898 | . 9901 | . 9904 | . 9906 | . 9909 | . 9911 | . 9913 | . 9916 |
| 2.4 | . 9918 | . 9920 | . 9922 | . 9925 | . 9927 | . 9929 | . 9931 | . 9932 | . 9934 | . 9936 |
| 2.5 | . 9938 | . 9940 | . 9941 | . 9943 | . 9945 | . 9946 | . 9948 | . 9949 | . 9951 | . 9952 |
| 2.6 | . 9953 | . 9955 | . 9956 | . 9957 | . 9959 | . 9960 | . 9961 | . 9962 | . 9963 | . 9964 |
| 2.7 | . 9965 | . 9966 | . 9967 | . 9968 | . 9969 | . 9970 | . 9971 | . 9972 | . 9973 | . 9974 |
| 2.8 | . 9974 | . 9975 | . 9976 | . 9977 | . 9977 | . 9978 | . 9979 | . 9979 | . 9980 | . 9981 |
| 2.9 | . 9981 | . 9982 | . 9982 | . 9983 | . 9984 | . 9984 | . 9985 | . 9985 | . 9986 | . 9986 |
| 3.0 | . 9987 | . 9987 | . 9987 | . 9988 | . 9988 | . 9989 | . 9989 | . 9989 | . 9990 | . 9990 |
| 3.1 | . 9990 | . 9991 | . 9991 | . 9991 | . 9992 | . 9992 | . 9992 | . 9992 | . 9993 | . 9993 |
| 3.2 | . 9993 | . 9993 | . 9994 | . 9994 | . 9994 | . 9994 | . 9994 | . 9995 | . 9995 | . 9995 |
| 3.3 | . 9995 | . 9995 | . 9995 | . 9996 | . 9996 | . 9996 | . 9996 | . 9996 | . 9996 | . 9997 |
| 3.4 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9998 |
| 3.5 | . 9998 | . 9998 | . 9998 | . 9998 | . 9998 | . 9998 | . 9998 | . 9998 | . 9998 | . 9998 |
| 3.6 | . 9998 | . 9998 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 |
| 3.7 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 |
| 3.8 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 |
| 3.9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

LOGARITHMS

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Mean Differences |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 000 | 0 | 008 | 0128 | 0170 | 0212 | 0 | 0294 | 03 | 0374 | 4 | 8 | 12 | 17 | 21 | 25 | 29 | 33 | 37 |
| 11 | 041 | 045 | 049 | 05 | 0569 | 06 | 0645 | 0682 | 0719 | 0755 |  | 8 | 11 | 15 | 19 | 23 | 2 | 30 | 34 |
| 12 | 0792 | 0828 | 0864 | 0899 | 093 | 0969 | 100 | 103 | 1072 | 11 | 3 | 7 | 10 | 14 | 17 | 21 | 2 | 28 | 31 |
| 13 | 1139 | 1173 | 1206 | 1239 | 1271 | 1303 | 1335 | 1367 | 1399 | 1430 | 3 | 6 | 10 | 13 | 16 | 19 | 2 | 26 | 29 |
| 14 | 1461 | 1492 | 1523 | 1553 | 15 | 1614 | 1644 | 1673 | 1703 | 1732 | 3 | 6 | 9 | 12 | 15 | 18 | 2 | 24 | 27 |
| 15 | 1761 | 1790 | 1818 | 1847 | 187 | 1903 | 193 | 195 | 1987 | 20 | 3 | 6 | 8 | 11 | 14 | 17 | 2 | 22 | 25 |
| 16 | 204 | 2068 | 209 | 2122 | 214 | 2175 | 220 | 222 | 225 | 22 | 3 | 5 | 8 | 11 | 13 | 6 | 18 | 21 | 24 |
| 17 | 2304 | 2330 | 2355 | 2380 | 2405 | 2430 | 2455 | 2480 | 2504 | 2529 | 2 | 5 | 7 | 10 | 12 | 15 | 17 | 20 | 22 |
| 18 | 2553 | 2577 | 2601 | 2625 | 2648 | 2672 | 2695 | 271 | 2742 | 2765 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 19 | 21 |
| 19 | 2788 | 2810 | 2833 | 2856 | 2878 | 2900 | 2923 | 2945 | 2967 | 2989 | 2 | 4 | 7 | 9 | 11 | 3 | 1 | 18 | 20 |
| 20 | 3010 | 3032 | 3054 | 3075 | 3096 | 3118 | 3139 | 3160 | 3181 | 3201 | 2 | 4 | 6 | 8 | 11 | 13 | 1 | 17 | 19 |
| 21 | 3222 | 3243 | 3263 | 3284 | 3304 | 3324 | 3345 | 3365 | 3385 | 34 | 2 |  | 6 | 8 | 10 | 12 |  | 16 | 18 |
| 22 | 3424 | 34 | 346 | 3483 | 3502 | 3522 | 35 | 35 | 35 | 3 | 2 |  | 6 | 8 | 10 | 12 |  | 15 | 17 |
| 23 | 3617 | 3636 | 3655 |  | 36 | 3711 | 3729 | 3747 | 37 |  | 2 |  | 6 | 7 |  | 11 |  | 15 | 17 |
| 24 | 3802 | 3820 | 3838 | 3856 | 387 | 3892 | 390 | 39 | 39 | 3962 | 2 |  | 5 | 7 |  | 11 |  | 1 | 16 |
| 25 | 3979 | 3997 | 401 | 403 | 40 | 4065 | 40 | 409 | 411 | 41 | 2 | 3 | 5 | 7 |  | 10 |  |  | 15 |
| 26 | 415 | 41 | 41 | 4 | 42 | 423 | 42 | 426 | 42 | 42 | 2 | 3 | 5 | 7 | 8 | 0 | 11 | 13 | 15 |
| 27 | 4314 | 4330 | 4346 | 43 | 4378 | 4393 | 4409 | 442 | 444 | 44 | 2 | 3 | 5 | 6 |  | 9 | 1 | 13 | 14 |
| 28 | 4472 | 4487 | 4502 | 4518 | 45 | 4548 | 45 | 45 | 45 | 46 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 12 | 14 |
| 2 | 4624 | 4639 | 465 | 4669 | 468 | 4698 | 47 | 4 | 474 | 47 |  | 3 | 4 | 6 | 7 | 9 | 10 |  | 3 |
| 30 | 47 | 47 | 4800 | 48 | 48 | 48 | 48 | 48 | 48 | 49 |  |  | 4 | 6 | 7 | 9 | 1 |  | 13 |
| 31 | 49 | 4 | 4942 | 49 | 49 | 4983 | 499 | 50 | 502 | 503 |  |  | 4 |  | 7 | 8 | 10 |  | 12 |
| 32 | 505 | 5065 | 5079 | 5092 | 51 | 511 | 51 | 5145 | 515 | 5172 |  |  | 4 | 5 | 7 | 8 |  | 11 | 2 |
| 33 | 518 | 51 | 521 | 52 | 52 | 525 | 526 | 52 | 528 | 53 |  |  | 4 |  |  | 8 |  |  | 12 |
| 34 | 53 | 5328 | 5340 | 53 | 53 | 53 | 5 | 5403 | 54 | 5428 |  |  | 4 |  |  | 8 |  |  | 11 |
| 35 | 5441 | 5453 | 5465 | 5478 | 54 | 5502 |  | 55 | 553 | 55 |  |  | 4 | 5 |  | 7 |  |  | 11 |
| 36 | 5563 | 5575 | 5587 | 5599 | 5611 | 5623 | 563 | 5 | 565 |  |  |  | 4 |  |  | 7 |  | 10 | 1 |
| 37 | 56 | 56 | 5705 | 57 | 57 | 57 | 57 | 5763 | 57 | 5786 |  |  | 3 |  |  | 7 |  |  | 10 |
| 38 | 5798 | 5809 | 5821 | 5832 | 584 | 585 | 58 | 58 | 588 |  |  |  | 3 |  |  | 7 |  |  | 10 |
| 39 | 591 | 5922 | 5933 | 5944 | 5955 | 5966 | 5977 | 598 | 59 |  |  |  | 3 |  |  | 7 |  |  | 10 |
| 40 | 602 | 6031 | 6042 | 605 | 60 | 607 | 60 | 609 | 61 | 6 |  |  | 3 | 4 |  | 6 |  |  | 10 |
| 41 | 6128 | 61 | 61 | 6 | 61 | 61 | 6191 | 6201 | 62 | 6 |  |  | 3 |  |  | 6 |  |  | 9 |
| 42 | 6232 | 6243 | 6253 | 6263 | 62 | 6284 | 6294 | 630 | 631 | 63 |  |  | 3 |  |  | 6 |  |  | 9 |
| 43 | 6335 | 6345 | 6355 | 6365 | 63 | 6385 | 6395 | 640 | 641 | 6 |  |  | 3 |  |  |  |  |  | 9 |
| 44 | 6435 | 6444 | 6454 | 6464 | 64 | 648 | 6493 | 65 | 65 |  |  |  | 3 |  |  |  |  |  | 9 |
| 45 | 6532 | 6542 | 6551 | 65 |  | 65 | 65 | 65 | 660 | 66 |  |  | 3 |  |  | 6 |  |  | 9 |
| 46 | 662 | 66 | 66 | 66 | 66 | 66 | 66 | 6693 | 67 |  |  |  | 3 |  |  | 6 |  |  | 8 |
| 47 | 6721 | 6730 | 6739 | 6749 | 67 | 6767 | 6776 | 6785 | 679 | 6803 |  |  | 3 |  |  | 5 |  |  | 8 |
| 48 | 6812 | 6821 | 6830 | 6839 | 684 | 68 | 6866 | 68 | 688 |  |  |  | 3 |  |  | 5 |  |  | 8 |
| 49 | 6902 | 6911 | 6920 | 6928 | 693 | 6946 | 6955 | 6964 | 697 | 6981 |  |  | 3 |  |  | 5 |  |  | 8 |
| 50 | 6990 | 6998 | 7007 | 7016 | 702 | 7033 | 7042 | 70 | 705 | 70 |  |  | 3 |  |  | 5 |  |  | 8 |
| 51 | 7076 | 7084 | 7093 | 7101 | 7110 | 7118 | 7126 | 7135 | 7143 | 7152 |  |  | 3 |  |  | 5 |  |  | 8 |
| 52 | 7160 | 7168 | 7177 | 7185 | 7193 | 7202 | 7210 | 7218 | 7226 | 7235 | 1 | 2 | 2 | 3 |  | 5 |  |  | 7 |
| 5 | 7243 | 7251 | 7259 | 7267 | 727 | 728 | 7292 | 7300 | 7308 | 73 |  |  | 2 |  |  | 5 |  |  | 7 |
| 54 | 7324 | 7332 | 7340 | 7348 | 7356 | 7364 | 7372 | 7380 | 7388 | 7396 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 6 | 7 |

LOGARITHMS

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Mean Differences |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 55 | 7404 | 7412 | 7419 | 7427 | 7435 | 7443 | 7451 | 7459 | 7466 | 7474 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 56 | 7482 | 7490 | 7497 | 7505 | 7513 | 7520 | 7528 | 7536 | 7543 | 7551 |  | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 57 | 7559 | 7566 | 7574 | 7582 | 7589 | 7597 | 7604 | 7612 | 7619 | 7627 |  | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 58 | 7634 | 7642 | 7649 | 7657 | 7664 | 7672 | 7679 | 7686 | 7694 | 7701 |  | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |
| 59 | 7709 | 7716 | 7723 | 7731 | 7738 | 7745 | 7752 | 7760 | 7767 | 7774 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |
| 60 | 7782 | 7789 | 7796 | 7803 | 7810 | 7818 | 7825 | 7832 | 7839 | 7846 |  | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 6 | 7853 | 7860 | 7868 | 7875 | 7882 | 7889 | 7896 | 7903 | 7910 | 7917 |  | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 62 | 7924 | 7931 | 7938 | 7945 | 7952 | 7959 | 7966 | 7973 | 7980 | 7987 |  | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 6 |
| 63 | 7993 | 8000 | 8007 | 8014 | 8021 | 8028 | 8035 | 8041 | 8048 | 8055 |  | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 64 | 8062 | 8069 | 8075 | 8082 | 8089 | 8096 | 8102 | 8109 | 8116 | 8122 |  | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 65 | 8129 | 8136 | 8142 | 8149 | 8156 | 8162 | 8169 | 8176 | 8182 | 8189 |  | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 66 | 8195 | 8202 | 8209 | 8215 | 8222 | 8228 | 8235 | 8241 | 8248 | 8254 |  |  | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 67 | 8261 | 8267 | 8274 | 8280 | 8287 | 8293 | 8299 | 8306 | 8312 | 8319 |  | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 68 | 8325 | 8331 | 8338 | 8344 | 8351 | 8357 | 8363 | 8370 | 8376 | 8382 |  | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 6 |
| 69 | 8388 | 8395 | 8401 | 8407 | 8414 | 8420 | 8426 | 8432 | 8439 | 8445 |  |  | 2 | 2 | 3 | 4 | 4 | 5 | 6 |
| 70 | 8451 | 8457 | 8463 | 8470 | 8476 | 8482 | 8488 | 8494 | 8500 | 8506 |  | 1 | 2 | 2 | 3 | 4 | 4 | 5 |  |
| 71 | 8513 | 8519 | 8525 | 8531 | 8537 | 8543 | 8549 | 8555 | 8561 | 8567 |  |  | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 72 | 8573 | 8579 | 8585 | 8591 | 8597 | 8603 | 8609 | 8615 | 8621 | 8627 |  | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 73 | 8633 | 8639 | 8645 | 8651 | 8657 | 8663 | 8669 | 8675 | 868 | 8686 |  |  | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 74 | 8692 | 8698 | 8704 | 8710 | 8716 | 8722 | 8727 | 8733 | 8739 | 8745 |  |  | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 75 | 8751 | 8756 | 8762 | 8768 | 8774 | 8779 | 8785 | 879 | 8797 | 8802 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 |
| 76 | 8808 | 8814 | 8820 | 8825 | 8831 | 8837 | 8842 | 8848 | 8854 | 8859 |  |  | 2 | 2 | 3 | 3 | 4 | 5 | 5 |
| 77 | 8865 | 8871 | 8876 | 8882 | 8887 | 8893 | 8899 | 8904 | 8910 | 8915 |  | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 78 | 8921 | 8927 | 8932 | 8938 | 8943 | 8949 | 8954 | 8960 | 8965 | 8971 |  |  | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 79 | 8976 | 8982 | 8987 | 8993 | 8998 | 9004 | 9009 | 9015 | 9020 | 9025 |  | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 80 | 9031 | 9036 | 9042 | 9047 | 9053 | 9058 | 9063 | 9069 | 9074 | 9079 |  | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 81 | 9085 | 9090 | 9096 | 9101 | 9106 | 9112 | 9117 | 9122 | 9128 | 9133 |  |  | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 82 | 9138 | 9143 | 9149 | 9154 | 9159 | 9165 | 9170 | 9175 | 9180 | 9186 |  |  | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 83 | 9191 | 9196 | 9201 | 9206 | 9212 | 9217 | 9222 | 9227 | 9232 | 9238 |  |  | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 84 | 9243 | 9248 | 9253 | 9258 | 9263 | 9269 | 9274 | 9279 | 9284 | 9289 |  |  | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 85 | 9294 | 9299 | 9304 | 9309 | 9315 | 9320 | 9325 | 9330 | 9335 | 9340 |  |  | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 86 | 9345 | 9350 | 9355 | 9360 | 9365 | 9370 | 9375 | 9380 | 9385 | 9390 |  |  | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 87 | 9395 | 9400 | 9405 | 9410 | 9415 | 9420 | 9425 | 9430 | 9435 | 9440 |  |  | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 88 | 9445 | 9450 | 9455 | 9460 | 9465 | 9469 | 9474 | 9479 | 9484 | 9489 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 89 | 9494 | 9499 | 9504 | 9509 | 9513 | 9518 | 9523 | 9528 | 9533 | 9538 |  |  | 1 | 2 | 2 | 3 | 3 | 4 |  |
| 90 | 9542 | 9547 | 9552 | 9557 | 9562 | 9566 | 9571 | 9576 | 958 | 958 | 0 |  | 1 | 2 | 2 | 3 | 3 | 4 |  |
| 91 | 9590 | 9595 | 9600 | 9605 | 9609 | 9614 | 9619 | 9624 | 9628 | 9633 |  |  | 1 | 2 | 2 | 3 | 3 | 4 |  |
| 92 | 9638 | 9643 | 9647 | 9652 | 9657 | 9661 | 9666 | 9671 | 9675 | 9680 |  |  | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 93 | 9685 | 9689 | 9694 | 9699 | 9703 | 9708 | 9713 | 9717 | 9722 | 9727 |  |  | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 94 | 9731 | 9736 | 9741 | 9745 | 9750 | 9754 | 9759 | 9763 | 9768 | 9773 |  |  | , | 2 | 2 | 3 | 3 | 4 | 4 |
| 95 | 9777 | 9782 | 9786 | 9791 | 9795 | 9800 | 9805 | 9809 | 9814 | 9818 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 96 | 9823 | 9827 | 9832 | 9836 | 9841 | 9845 | 9850 | 9854 | 9859 | 9863 |  |  | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 97 | 9868 | 9872 | 9877 | 9881 | 9886 | 9890 | 9894 | 9899 | 9903 | 9908 |  |  | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 98 | 9912 | 9917 | 9921 | 9926 | 9930 | 9934 | 9939 | 9943 | 9948 | 9952 |  |  | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 99 | 9956 | 9961 | 9965 | 9969 | 9974 | 9978 | 9983 | 9987 | 9991 | 9996 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 |


| ANTILOGARITHMS |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Mean Differences |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| . 00 | 1000 | 1002 | 1005 | 10 | 100 | 10 | 10 | 1016 | 1019 | 10 | 0 | 0 | 1 |  |  | 1 | 2 | 2 | 2 |
|  | 1023 | 1026 | 1028 | 1030 | 1033 | 1035 | 1038 | 1040 | 1042 | 1045 |  | 0 | 1 |  |  | 1 | 2 | 2 | 2 |
| . 02 | 1047 | 1050 | 1052 | 1054 | 1057 | 1059 | 1062 | 1064 | 1067 | 1069 | 0 | 0 | 1 |  |  | 1 | 2 | 2 | 2 |
| . 03 | 1072 | 1074 | 1076 | 1079 | 1081 | 1084 | 1086 | 1089 | 1091 | 1094 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| . 04 | 1096 | 1099 | 1102 | 1104 | 1107 | 1109 | 1112 | 1114 | 1117 | 1119 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| . 05 | 1122 | 1125 | 1127 | 1130 | 1132 | 1135 | 1138 | 1140 | 1143 | 1146 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| . 06 | 1148 | 1151 | 1153 | 1156 | 1159 | 1161 | 1164 | 1167 | 1169 | 1172 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| . 07 | 1175 | 1178 | 1180 | 1183 | 1186 | 1189 | 1191 | 1194 | 1197 | 1199 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| . 08 | 1202 | 1205 | 1208 | 1211 | 1213 | 1216 | 1219 | 1222 | 1225 | 1227 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 3 |
| . 09 | 1230 | 1233 | 1236 | 1239 | 1242 | 1245 | 1247 | 1250 | 1253 | 1256 | 0 | 1 | 1 | 1 |  | 2 | 2 | 2 | 3 |
| . 10 | 1259 | 1262 | 1265 | 1268 | 1271 | 1274 | 1276 | 1279 | 1282 | 1285 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 3 |
| . 11 | 1288 | 1291 | 1294 | 1297 | 1300 | 1303 | 1306 | 1309 | 1312 | 1315 |  |  | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| . 12 | 1318 | 1321 | 1324 | 1327 | 1330 | 1334 | 1337 | 1340 | 1343 | 1346 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| . 13 | 1349 | 1352 | 1355 | 1358 | 1361 | 1365 | 1368 | 1371 | 1374 | 1377 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| . 14 | 1380 | 1384 | 1387 | 1390 | 1393 | 1396 | 1400 | 1403 | 1406 | 1409 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| . 15 | 1413 | 1416 | 1419 | 1422 | 1426 | 1429 | 1432 | 1435 | 1439 | 1442 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| . 16 | 1445 | 1449 | 1452 | 1455 | 1459 | 1462 | 1466 | 1469 | 1472 | 1476 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| . 17 | 1479 | 1483 | 1486 | 1489 | 1493 | 1496 | 1500 | 1503 | 1507 | 1510 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| . 18 | 1514 | 1517 | 1521 | 1524 | 1528 | 1531 | 1535 | 1538 | 1542 | 1545 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| . 19 | 1549 | 1552 | 1556 | 1560 | 1563 | 1567 | 1570 | 1574 | 1578 | 1581 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 |
| . 20 | 1585 | 1589 | 1592 | 1596 | 1600 | 1603 | 1607 | 1611 | 1614 | 1618 | 0 | 1 | 1 | 1 | 2 | 2 | 3 | 3 | 3 |
| . 21 | 1622 | 1626 | 1629 | 1633 | 1637 | 1641 | 1644 | 1648 | 1652 | 1656 |  | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 |
| . 22 | 1660 | 1663 | 1667 | 1671 | 1675 | 1679 | 1683 | 1687 | 1690 | 1694 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 |
| . 23 | 1698 | 1702 | 1706 | 1710 | 1714 | 1718 | 1722 | 1726 | 1730 | 1734 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 4 |
| . 24 | 1738 | 1742 | 1746 | 1750 | 1754 | 1758 | 1762 | 1766 | 1770 | 1774 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 4 |
| . 25 | 1778 | 1782 | 1786 | 1791 | 1795 | 1799 | 1803 | 1807 | 1811 | 1816 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 4 |
| . 26 | 1820 | 1824 | 1828 | 1832 | 1837 | 1841 | 1845 | 1849 | 1854 | 1858 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 |
| . 27 | 1862 | 1866 | 1871 | 1875 | 1879 | 1884 | 1888 | 1892 | 1897 | 1901 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 |
| . 28 | 1905 | 1910 | 1914 | 1919 | 1923 | 1928 | 1932 | 1936 | 1941 | 1945 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| . 29 | 1950 | 1954 | 1959 | 1963 | 1968 | 1972 | 1977 | 1982 | 1986 | 1991 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| . 30 | 1995 | 2000 | 2004 | 2009 | 2014 | 2018 | 2023 | 2028 | 2032 | 2037 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| . 31 | 2042 | 2046 | 2051 | 2056 | 2061 | 2065 | 2070 | 2075 | 2080 | 2084 |  |  | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| . 32 | 2089 | 2094 | 2099 | 2104 | 2109 | 2113 | 2118 | 2123 | 2128 | 2133 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| . 33 | 2138 | 2143 | 2148 | 2153 | 2158 | 2163 | 2168 | 2173 | 2178 | 2183 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| . 34 | 2188 | 2193 | 2198 | 2203 | 2208 | 2213 | 2218 | 2223 | 2228 | 2234 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| . 35 | 2239 | 2244 | 2249 | 2254 | 2259 | 2265 | 2270 | 2275 | 2280 | 2286 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| . 36 | 2291 | 2296 | 2301 | 2307 | 2312 | 2317 | 2323 | 2328 | 2333 | 2339 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| . 37 | 2344 | 2350 | 2355 | 2360 | 2366 | 2371 | 2377 | 2382 | 2388 | 2393 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| . 38 | 2399 | 2404 | 2410 | 2415 | 2421 | 2427 | 2432 | 2438 | 2443 | 2449 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| . 39 | 2455 | 2460 | 2466 | 2472 | 2477 | 2483 | 2489 | 2495 | 2500 | 2506 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 |
| . 40 | 2512 | 2518 | 2523 | 2529 | 2535 | 2541 | 2547 | 2553 | 2559 | 2564 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| . 41 | 2570 | 2576 | 2582 | 2588 | 2594 | 2600 | 2606 | 2612 | 2618 | 2624 |  | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| . 42 | 2630 | 2636 | 2642 | 2649 | 2655 | 2661 | 2667 | 2673 | 2679 | 2685 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 |
| . 43 | 2692 | 2698 | 2704 | 2710 | 2716 | 2723 | 2729 | 2735 | 2742 | 2748 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 6 |
| . 44 | 2754 | 2761 | 2767 | 2773 | 2780 | 2786 | 2793 | 2799 | 2805 | 2812 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 6 |
| . 45 | 2818 | 2825 | 2831 | 2838 | 2844 | 2851 | 2858 | 2864 | 2871 | 2877 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| . 46 | 2884 | 2891 | 2897 | 2904 | 2911 | 2917 | 2924 | 2931 | 2938 | 2944 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| . 47 | 2951 | 2958 | 2965 | 2972 | 2979 | 2985 | 2992 | 2999 | 3006 | 3013 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| . 48 | 3020 | 3027 | 3034 | 3041 | 3048 | 3055 | 3062 | 3069 | 3076 | 3083 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| . 49 | 3090 | 3097 | 3105 | 3112 | 3119 | 3126 | 3133 | 3141 | 3148 | 3155 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |


| ANTILOGARITHMS |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{\|l\|} \hline{ }_{4}^{\circ} \\ \hline 8 \end{array}$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Mean Differences |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  | 1 | 2 | 3 |  | 5 | 6 | 7 | 8 | 9 |
| . 50 | 3162 | 3170 | 3177 | 318 | 3192 | 3199 | 3206 | 321 | 322 | 322 |  | 1 | 2 |  | 4 | 4 | 5 |  | 7 |
| . 51 | 3236 | 3243 | 3251 | 3258 | 3266 | 3273 | 3281 | 3289 | 3296 | 3304 |  | 2 | 2 |  | 4 | 5 | 5 | 6 | 7 |
| . 52 | 3311 | 3319 | 3327 | 3334 | 3342 | 3350 | 3357 | 3365 | 3373 | 3381 |  | 2 | 2 |  | 4 | 5 | 5 | 6 | 7 |
| . 53 | 3388 | 3396 | 3404 | 3412 | 3420 | 3428 | 3436 | 3443 | 3451 | 3459 | 1 | 2 | 2 |  | 4 | 5 | 6 | 6 | 7 |
| . 54 | 3467 | 3475 | 3483 | 3491 | 3499 | 3508 | 3516 | 3524 | 3532 | 3540 | 1 | 2 | 2 |  | 4 | 5 | 6 | 6 | 7 |
| . 55 | 3548 | 3556 | 3565 | 3573 | 3581 | 3589 | 3597 | 3606 | 3614 | 3622 | 1 | 2 | 2 |  | 4 | 5 | 6 | 7 | 7 |
| . 56 | 3631 | 3639 | 3648 | 3656 | 3664 | 3673 | 3681 | 3690 | 3698 | 3707 | 1 | 2 | 3 |  | 4 | 5 | 6 | 7 | 8 |
| . 57 | 3715 | 3724 | 3733 | 3741 | 3750 | 3758 | 3767 | 3776 | 3784 | 3793 | 1 | 2 | 3 |  | 4 | 5 | 6 | 7 | 8 |
| . 58 | 3802 | 3811 | 3819 | 3828 | 3837 | 3846 | 3855 | 3864 | 3873 | 3882 | 1 | 2 | 3 |  | 4 | 5 | 6 | 7 | 8 |
| . 59 | 3890 | 3899 | 3908 | 3917 | 3926 | 3936 | 3945 | 3954 | 3963 | 3972 | 1 | 2 | 3 |  | 5 | 5 | 6 | 7 | 8 |
| . 60 | 3981 | 3990 | 3999 | 4009 | 4018 | 4027 | 4036 | 4046 | 4055 | 4064 | 1 | 2 | 3 |  | 5 | 6 | 6 | 7 | 8 |
| . 61 | 4074 | 4083 | 4093 | 4102 | 4111 | 4121 | 4130 | 4140 | 4150 | 4159 |  | 2 | 3 |  | 5 | 6 | 7 | 8 | 9 |
| . 62 | 4169 | 4178 | 4188 | 4198 | 4207 | 4217 | 4227 | 4236 | 4246 | 4256 | 1 | 2 | 3 |  | 5 | 6 | 7 | 8 | 9 |
| . 63 | 4266 | 4276 | 4285 | 4295 | 4305 | 4315 | 4325 | 4335 | 4345 | 4355 | 1 | 2 | 3 |  | 5 | 6 | 7 | 8 | 9 |
| . 64 | 4365 | 4375 | 4385 | 4395 | 4406 | 4416 | 4426 | 4436 | 4446 | 4457 | 1 | 2 | 3 |  | 5 | 6 | 7 | 8 | 9 |
| . 65 | 4467 | 4477 | 4487 | 4498 | 4508 | 4519 | 4529 | 4539 | 4550 | 4560 | 1 | 2 | 3 |  | 5 | 6 | 7 | 8 | 9 |
| . 66 | 4571 | 4581 | 4592 | 4603 | 4613 | 4624 | 4634 | 4645 | 4656 | 4667 | 1 | 2 | 3 |  | 5 | 6 | 7 | 9 | 10 |
| . 67 | 4677 | 4688 | 4699 | 4710 | 4721 | 4732 | 4742 | 4753 | 4764 | 4775 | 1 | 2 | 3 |  | 5 | 7 | 8 | 9 | 10 |
| . 68 | 4786 | 4797 | 4808 | 4819 | 4831 | 4842 | 4853 | 4864 | 4875 | 4887 | 1 | 2 | 3 |  | 6 | 7 | 8 | 9 | 10 |
| . 69 | 4898 | 4909 | 4920 | 4932 | 4943 | 4955 | 4966 | 4977 | 4989 | 5000 | 1 | 2 | 3 |  | 6 | 7 | 8 | 9 | 10 |
| . 70 | 5012 | 5023 | 5035 | 5047 | 5058 | 5070 | 5082 | 5093 | 5105 | 5117 | 1 | 2 | 4 |  | 6 | 7 | 8 |  | 11 |
| . 7 | 5129 | 5140 | 5152 | 5164 | 5176 | 5188 | 5200 | 5212 | 5224 | 523 |  | 2 | 4 |  | 6 | 7 | 8 | 10 | 11 |
| . 72 | 5248 | 5260 | 5272 | 5284 | 5297 | 5309 | 5321 | 5333 | 5346 | 5358 | 1 | 2 | 4 |  | 6 | 7 | 9 | 10 | 11 |
| . 73 | 5370 | 5383 | 5395 | 5408 | 5420 | 5433 | 5445 | 5458 | 5470 | 5483 | 1 | 3 | 4 |  | 6 | 8 | 9 | 10 | 11 |
| . 74 | 5495 | 5508 | 5521 | 5534 | 5546 | 5559 | 5572 | 5585 | 5598 | 5610 | 1 | 3 | 4 |  | 6 | 8 | 9 | 10 | 12 |
| . 75 | 5623 | 5636 | 5649 | 5662 | 5675 | 5689 | 5702 | 5715 | 5728 | 574 | 1 | 3 | 4 | 5 | 7 | 8 | 9 | 10 | 12 |
| . 7 | 5754 | 5768 | 5781 | 5794 | 5808 | 5821 | 5834 | 5848 | 5861 | 5875 | 1 | 3 | 4 |  | 7 | 8 | 9 | 11 | 12 |
| . 77 | 5888 | 5902 | 5916 | 5929 | 5943 | 5957 | 5970 | 5984 | 5998 | 6012 | 1 | 3 | 4 |  | 7 | 8 | 10 | 11 | 12 |
| . 78 | 6026 | 6039 | 6053 | 6067 | 6081 | 6095 | 6109 | 6124 | 6138 | 6152 | 1 | 3 | 4 |  | 7 | 8 | 10 | 11 | 13 |
| . 79 | 6166 | 6180 | 6194 | 6209 | 6223 | 6237 | 6252 | 6266 | 6281 | 6295 | 1 | 3 | 4 |  | 7 | 9 | 10 | 11 | 13 |
| . 80 | 6310 | 6324 | 6339 | 6353 | 6368 | 6383 | 6397 | 6412 | 6427 | 6442 | 1 | 3 | 4 |  | 7 | 9 | 10 | 12 | 13 |
| . 81 | 6457 | 6471 | 6486 | 6501 | 6516 | 6531 | 6546 | 6561 | 6577 | 6592 | 2 | 3 | 5 |  | 8 | 9 | 11 | 12 | 14 |
| . 82 | 6607 | 6622 | 6637 | 6653 | 6668 | 6683 | 6699 | 6714 | 6730 | 6745 | 2 | 3 | 5 |  | 8 | 9 | 11 | 12 | 14 |
| . 83 | 6761 | 6776 | 6792 | 6808 | 6823 | 6839 | 6855 | 6871 | 6887 | 6902 | 2 | 3 | 5 |  | 8 | 9 | 11 | 13 | 14 |
| . 84 | 6918 | 6934 | 6950 | 6966 | 6982 | 6998 | 7015 | 7031 | 7047 | 7063 | 2 | 3 | 5 |  | 8 | 10 | 11 | 13 | 15 |
| . 85 | 7079 | 7096 | 7112 | 7129 | 7145 | 7161 | 7178 | 7194 | 7211 | 7228 | 2 | 3 | 5 |  | 8 | 10 | 12 | 13 | 15 |
| . 86 | 7244 | 7261 | 7278 | 729 | 7311 | 7328 | 7345 | 7362 | 7379 | 7396 | 2 | 3 | 5 |  | 8 | 10 | 12 | 13 | 15 |
| . 87 | 7413 | 7430 | 7447 | 7464 | 7482 | 7499 | 7516 | 7534 | 7551 | 7568 | 2 | 3 | 5 | 7 | 9 | 10 | 12 | 14 | 16 |
| . 88 | 7586 | 7603 | 7621 | 7638 | 7656 | 7674 | 7691 | 7709 | 7727 | 7745 | 2 | 4 | 5 |  | 9 | 11 | 12 | 14 | 16 |
| . 89 | 7762 | 7780 | 7798 | 7816 | 7834 | 7852 | 7870 | 7889 | 7907 | 7925 | 2 |  | 5 |  | 9 | 11 | 13 | 14 | 16 |
| . 90 | 7943 | 7962 | 7980 | 7998 | 8017 | 8035 | 8054 | 8072 | 8091 | 8110 | 2 | 4 | 6 |  | 9 | 11 | 13 | 15 | 17 |
| . 91 | 8128 | 8147 | 8166 | 8185 | 8204 | 8222 | 8241 | 8260 | 8279 | 8299 | 2 | 4 | 6 |  | 9 | 11 | 13 | 15 | 17 |
| . 92 | 8318 | 8337 | 8356 | 8375 | 8395 | 8414 | 8433 | 8453 | 8472 | 8492 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 15 | 17 |
| . 93 | 8511 | 8531 | 8551 | 8570 | 8590 | 8610 | 8630 | 8650 | 8670 | 8690 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| . 94 | 8710 | 8730 | 8750 | 8770 | 8790 | 8810 | 8831 | 8851 | 8872 | 8892 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| . 95 | 8913 | 8933 | 8954 | 8974 | 8995 | 9016 | 9036 | 9057 | 9078 | 9099 | 2 | 4 | 6 | 8 | 10 | 12 | 15 | 17 | 19 |
| . 96 | 9120 | 9141 | 9162 | 9183 | 9204 | 9226 | 9247 | 9268 | 9290 | 9311 | 2 | 4 | 6 |  | 11 | 13 | 15 | 17 | 19 |
| . 97 | 9333 | 9354 | 9376 | 9397 | 9419 | 9441 | 9462 | 9484 | 9506 | 9528 | 2 | 4 | 7 | 9 | 11 | 13 | 15 | 17 | 20 |
| . 98 | 9550 | 9572 | 9594 | 9616 | 9638 | 9661 | 9683 | 9705 | 9727 | 9750 | 2 | 4 | 7 |  | 11 | 13 | 16 | 18 | 20 |
| . 99 | 9772 | 9795 | 9817 | 9840 | 9863 | 9886 | 9908 | 9931 | 9954 | 9977 | 2 | 5 | 7 | 9 | 11 | 14 | 16 | 18 | 20 |

